Efficient dynamic resource allocation contracts in online platforms

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BACKGROUND

长沙货拉拉女乘客跳车事件 话题

简介：2021 年货拉拉乘客跳车事件，是 2021 年 2 月 6 日 21 时发生在湖南省长沙市岳麓区曲苑路的一起事件，造成一名乘客死亡。乘客车莎莎（1997 年或 1998 年-2021 年 2 月 10 日，23 岁）在货拉拉平台下单搬家货运服务，途中从副驾驶跳窗，后 更多.

最新讨论

货拉拉坠亡案司机首次面对镜头，称「她 4 次提出偏航，跳车时没反应过来」，还透露了哪些细节？

665 回答 · 508 万浏览

同工Same：同工我深夜来写一段话：货拉拉案件目前情况我懂，所以我这里不谈。我只是想分享一个情况给所有的机关、团体、项目负责人乃至所有网友。你但凡用过某博，你就能发现个人账户基本每条--博都有 付费推广的入口。 阅读全文

▲ 赞同 2387 ▼ 144 条评论 12-09

货拉拉案司机妻子发文质疑一审程序违法，法援强行占坑，并透露二审不开庭审理，如何从法律角度进行分析？

288 回答 · 191 万浏览

老刀把子：这个案子的社会效果出乎意料！长沙司法部门通过这个案子，给全国人民上了一堂生动的法治课。无数法学专家、律师，呼吁了这么多年程序正义，... 阅读全文

▲ 赞同 2196 ▼ 119 条评论 11-22

安徽 社会 外卖 校园热点 大学生辱骂外卖员

学校回应「安徽一女大学生辱骂外卖员致其辞职」事件，称希望给新生改错的机会，如何评价此事？

潇湘晨报，已认证账号

专题收录 知乎媒体每周看点1206～1212

最新：

近日，安徽合肥，一段女大学生辱骂外卖骑手的视频引网友关注。据了解，此单外卖是因顾客手机号码有误导致无法正常配送。再次配送时，骑手小猛希望该女生能说谢谢，结果引来辱骂。“在这打工就是我儿子、没钱就好好打工。”小猛称当时正在送餐，听到被骂后不知道该怎么办，“很无助，很屈辱”。

小猛表示受此事影响，无心工作，已辞职。他告诉记者，这件事情对他侮辱性太大了，为了几块钱的配送费，“打不还手，骂不还口。”“骑手再穷也靠努力挣钱的。如果谁家庭稍微好一点的话，谁会选择送外卖。”

对此，学校工作人员表示，当事女生已道歉且外卖员也接受了，并表示不会把视频上传到网上。学校也就此事联系了女生的家长。他表示，女孩是新生，学校希望网络舆论能对女孩宽容点，给她改正错误的机会。后续学校也会以此为戒在学校开展三观教育。

女大学生辱骂外卖员致其辞职 学校：希望给新生改错的机会
news.sina.cn/kx/2021-12-09/detail-ikyam...

INFORMATION ASYMMETRY

- Hidden Information --- adverse selection
- Hidden Action --- moral hazard

- Example: Health insurance:
  - Medical history
  - Body exercise
- Other examples: car insurance, tenure track…

Contract Theory
Mechanism Design

BACKGROUND---HOW TO DISSOLVE?

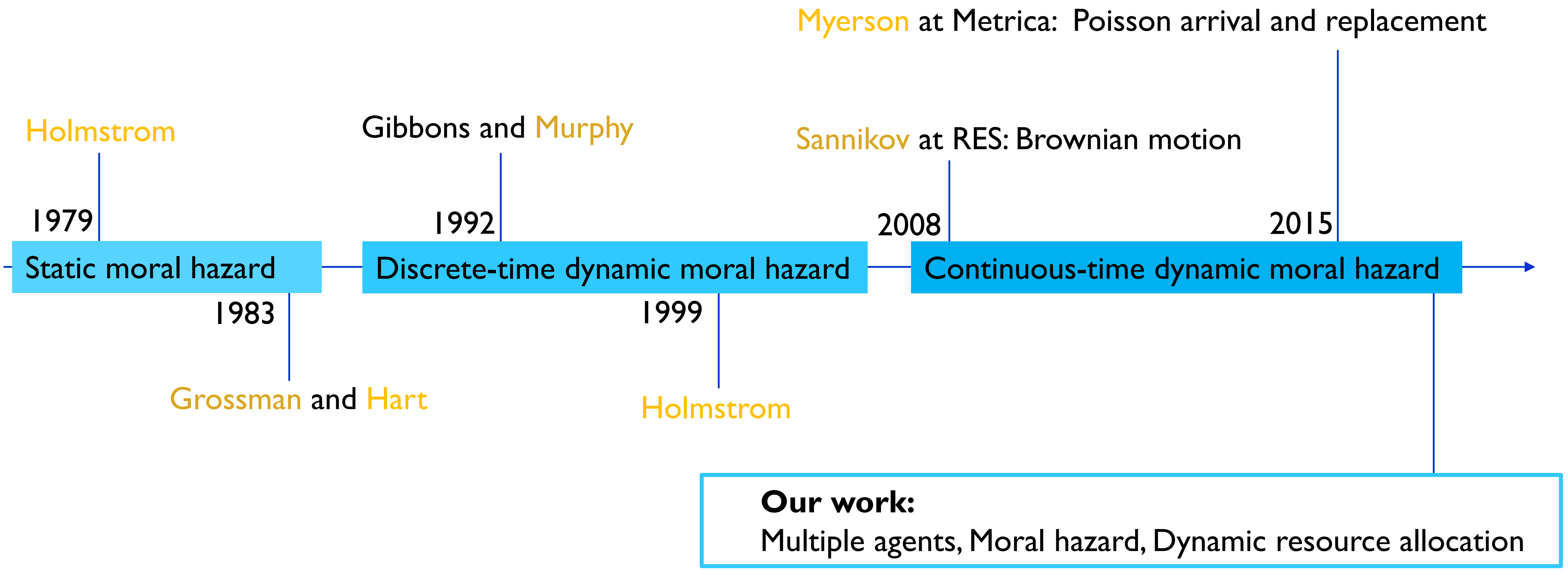
Platform	**Category**	**Examples of adverse events**	**Possible ramifications**
淘宝	Online shopping	Counterfeit goods	Search wight demotion
京东		Breach of promises	Limitation on promotion
		Fake transactions	Termination
滴滴	Ride hailing	Drugs and alcohol	Order dispatch reduction
		Safety issues	Suspension
		Fraud and theft	Termination
饿了么	Food delivery	Spoiled food	Subsidy reduction
美团		Wrong deliver	Suspension
			Termination
头条	UGC	Typos	Post restrictions
		Fake news	Limitation on promotion

RESEARCH QUESTION

How to design an optimal contract to induce continuous full effort?

- Multiple agents
- Continuous time
- Moral hazard
- Dynamic resource allocation

LITERATURE



Holmstrom

1979

Static moral hazard

1983

Grossman and Hart

Gibbons and Murphy

1992

Discrete-time dynamic moral hazard

1999

Holmstrom

Sannikov at RES: Brownian motion

2008

Myerson at Metrica: Poisson arrival and replacement

2015

Continuous-time dynamic moral hazard

Our work:
Multiple agents, Moral hazard, Dynamic resource allocation

MODEL

Principal (Platform)

- Payment: L Allocation: X
- Cost of bad arrival: CX Revenue: $R^p X$

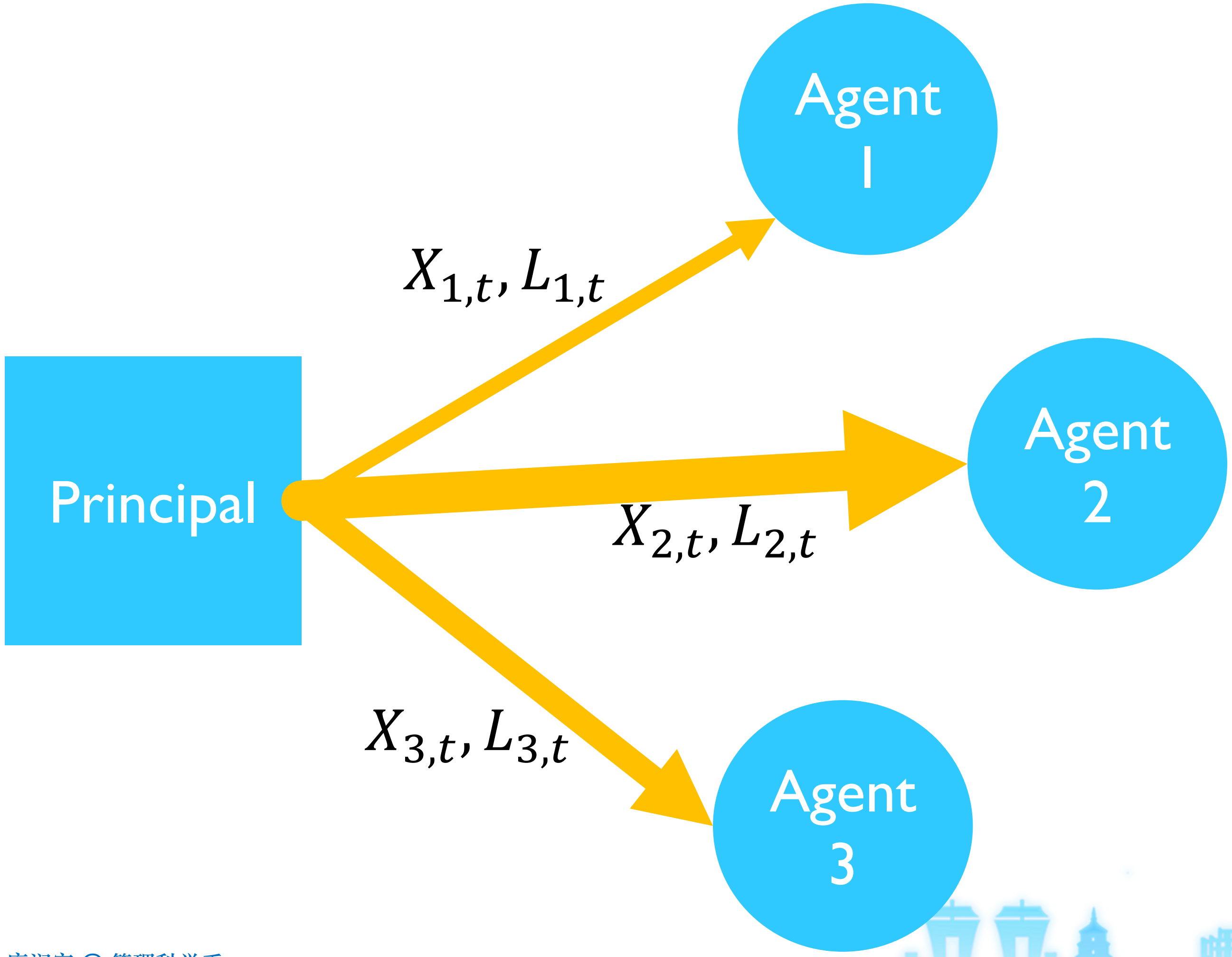
Profit: $$U(\Gamma) = E[\int_0^\infty e^{-\rho t} \sum (\underbrace{R^p X_{i,t} \mathrm{d}t}_{\text{Revenue}} - \overbrace{C X_{i,t} \mathrm{d}N_{i,t}}^{\text{Bad arrival cost}} \underbrace{- \mathrm{d}L_{i,t})}_{\text{Payment}}]$$

Agent (Supplier)

- Poisson arrival: with effort λ, without effort $\bar{\lambda}$. $(0 < \lambda < \bar{\lambda})$
- Revenue: $R^a X$ Shirking benefit: bX

Promised utility: $$W_{i,t} = E[\int_t^\infty e^{-\rho(\tau - t)} \sum (\underbrace{\mathrm{d}L_{i,\tau}}_{\text{Payment}} + \overbrace{R^a X_{i,t} \mathrm{d}\tau}^{\text{Reserved revenue}} + \underbrace{b X_{i,t} \mathbb{I}_{\lambda_\tau = \bar{\lambda}} \mathrm{d}\tau}_{\text{Shirking benefit}}]$$

MODEL



- Goal:

Find the optimal contract that maximize principal's total discounted profit.

$$\max_{\Gamma} \; U(\Gamma)$$

s.t. Incentive compatible (IC)

Incentive rationality (IR)

Promise keeping (PK)

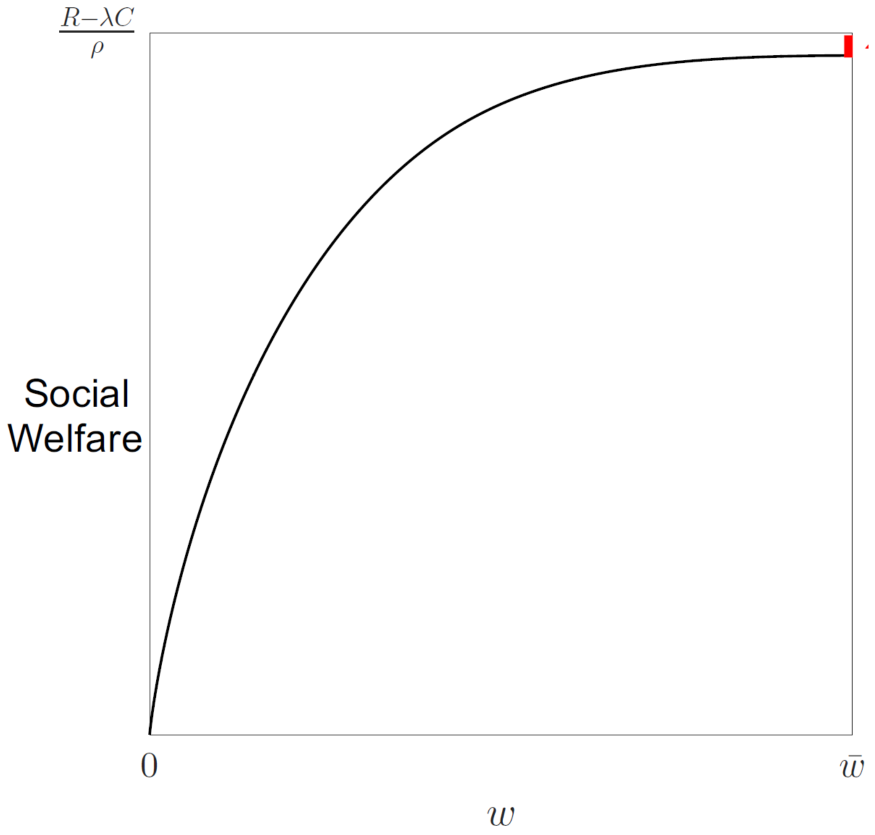
Limited liability (LL)

$$\Gamma = \{\boldsymbol{X}_t, \boldsymbol{L}_t\}_{t\geq 0}$$
$$= \{\boldsymbol{X}(\boldsymbol{W}_t), \boldsymbol{L}(\boldsymbol{W}_t)\}_{t\geq 0}$$

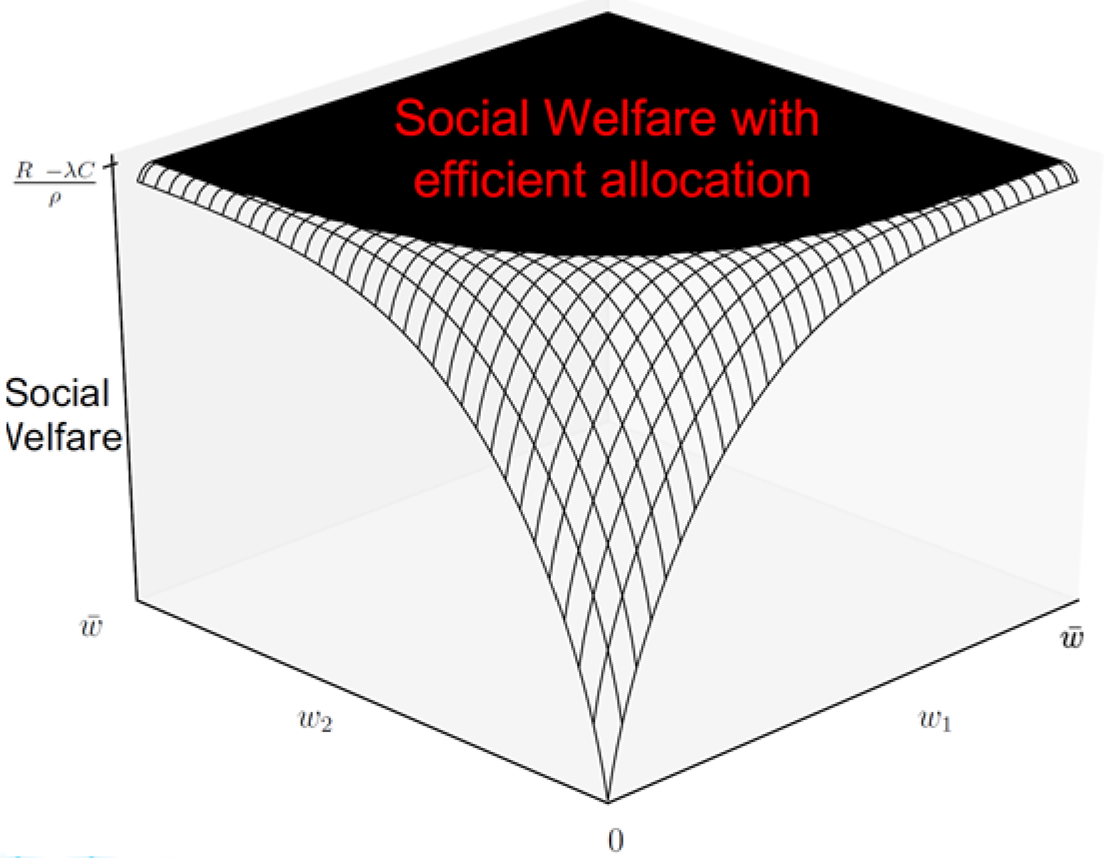
CONVENTIONAL APPROACH

$$S(\Gamma,\Lambda) = (R-\lambda C)E[\int_0^\infty e^{-\rho t}\sum_i X_{i,t}\,\mathrm{d}t]$$

Single-agent case



Two-agent case



Efficient and Incentive Compatible(EIC) contract

- Many platforms cannot reserve traffics.
  - E.g. 头条、饿了么…
- EIC contract: Induce all agents to exert efforts and achieves efficient allocation.

$$\sum X_{i,t} = 1, \qquad \forall t \geq 0.$$

$$\max_{\Gamma} \; U(\Gamma)$$

s.t. Incentive compatible (IC)
Incentive rationality (IR)
Promise keeping (PK)
Limited liability (LL)
Efficient allocation

SELF-GENERATING AND ACHIEVABLE SET

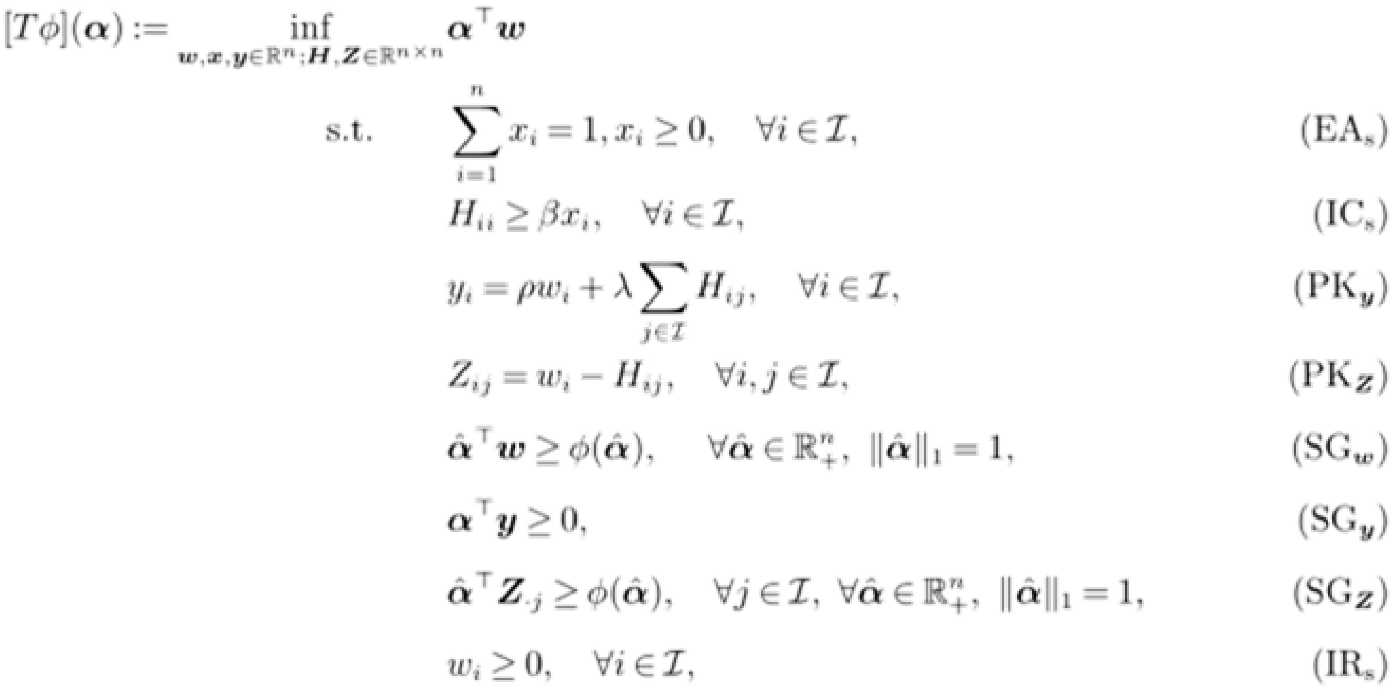
- **(Repeated Game)** A set is self-generating if every pay-off in the set is decomposable on the set.
- Extend to **continuous time:** Set A is self-generating, if for any $\boldsymbol{W}_0 \in A$, follow a EIC contract, $\boldsymbol{W}_t \in A$ for all $t \geq 0$.

Proposition: If set A is a self-generating set, then $A \in U$.

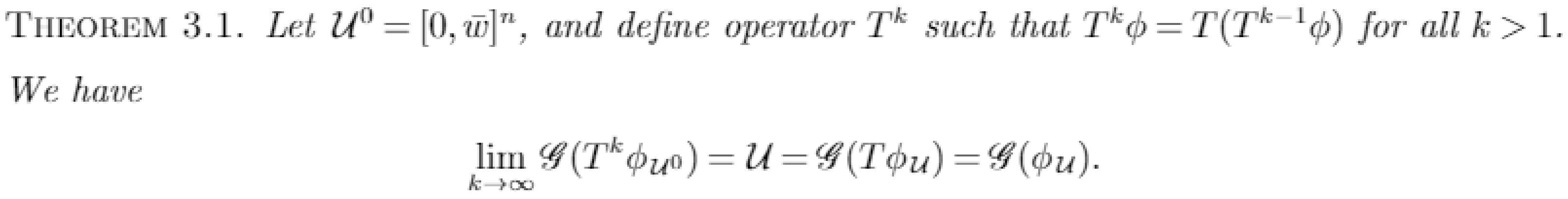
Proposition: The achievable set U is a self-generating set.

The achievable set U is the largest self-generating set.

Iterative Approach



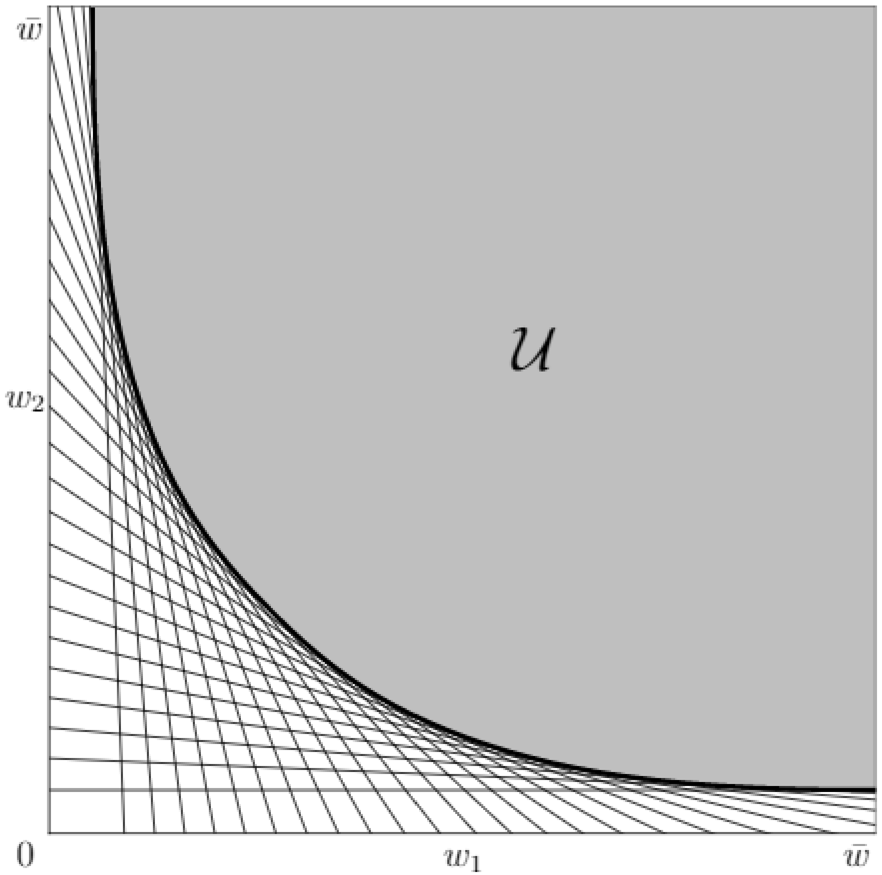
$$
\begin{array}{lll}
[T\phi](\boldsymbol{\alpha}) := \inf\limits_{\boldsymbol{w},\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}^n;\boldsymbol{H},\boldsymbol{Z}\in\mathbb{R}^{n\times n}} & \boldsymbol{\alpha}^\top \boldsymbol{w} & \\
\text{s.t.} & \sum_{i=1}^{n} x_i = 1, x_i \geq 0, \quad \forall i \in \mathcal{I}, & (\mathrm{EA_s}) \\
& H_{ii} \geq \beta x_i, \quad \forall i \in \mathcal{I}, & (\mathrm{IC_s}) \\
& y_i = \rho w_i + \lambda \sum_{j\in\mathcal{I}} H_{ij}, \quad \forall i \in \mathcal{I}, & (\mathrm{PK}_{\boldsymbol{y}}) \\
& Z_{ij} = w_i - H_{ij}, \quad \forall i,j \in \mathcal{I}, & (\mathrm{PK}_{\boldsymbol{Z}}) \\
& \hat{\boldsymbol{\alpha}}^\top \boldsymbol{w} \geq \phi(\hat{\boldsymbol{\alpha}}), \quad \forall \hat{\boldsymbol{\alpha}} \in \mathbb{R}^n_+, \ \|\hat{\boldsymbol{\alpha}}\|_1 = 1, & (\mathrm{SG}_{\boldsymbol{w}}) \\
& \boldsymbol{\alpha}^\top \boldsymbol{y} \geq 0, & (\mathrm{SG}_{\boldsymbol{y}}) \\
& \hat{\boldsymbol{\alpha}}^\top \boldsymbol{Z}_{\cdot j} \geq \phi(\hat{\boldsymbol{\alpha}}), \quad \forall j \in \mathcal{I}, \ \forall \hat{\boldsymbol{\alpha}} \in \mathbb{R}^n_+, \ \|\hat{\boldsymbol{\alpha}}\|_1 = 1, & (\mathrm{SG}_{\boldsymbol{Z}}) \\
& w_i \geq 0, \quad \forall i \in \mathcal{I}, & (\mathrm{IR_s})
\end{array}
$$



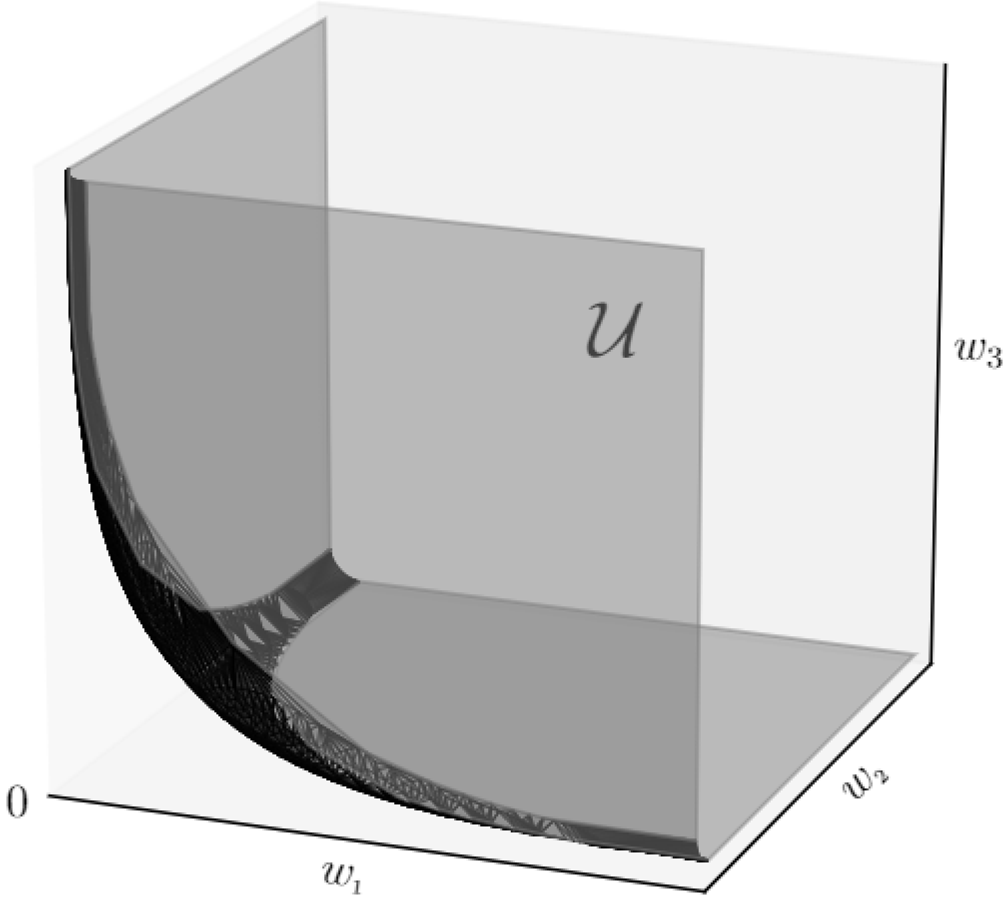
THEOREM 3.1. *Let $\mathcal{U}^0 = [0, \bar{w}]^n$, and define operator T^k such that $T^k\phi = T(T^{k-1}\phi)$ for all $k > 1$. We have*

$$\lim_{k\to\infty} \mathscr{G}(T^k \phi_{\mathcal{U}^0}) = \mathcal{U} = \mathscr{G}(T\phi_{\mathcal{U}}) = \mathscr{G}(\phi_{\mathcal{U}}).$$

ACHIEVABLE SET

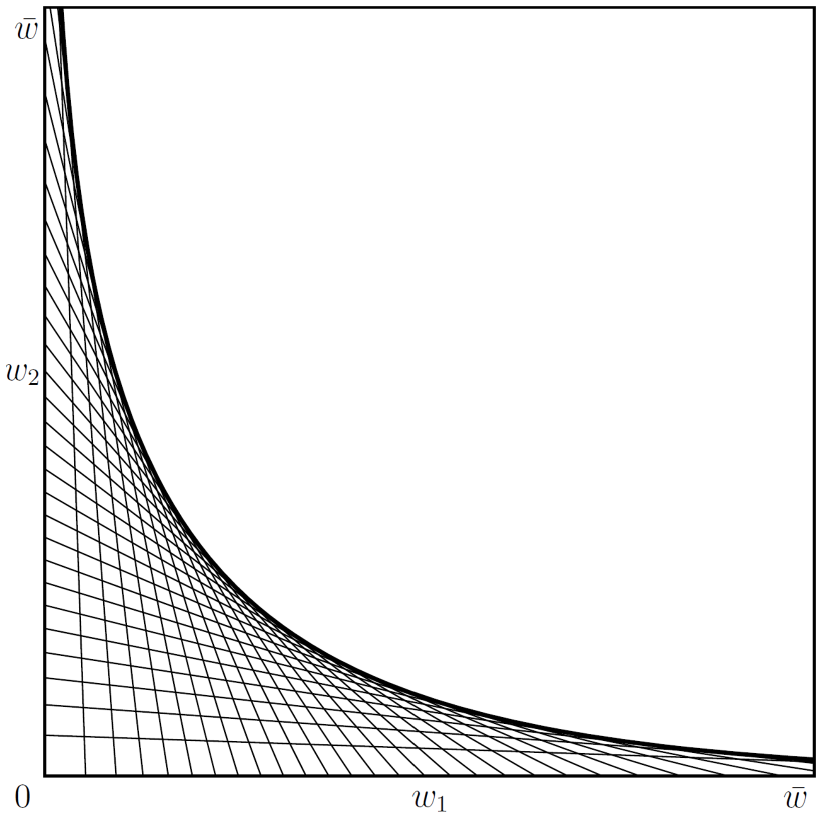


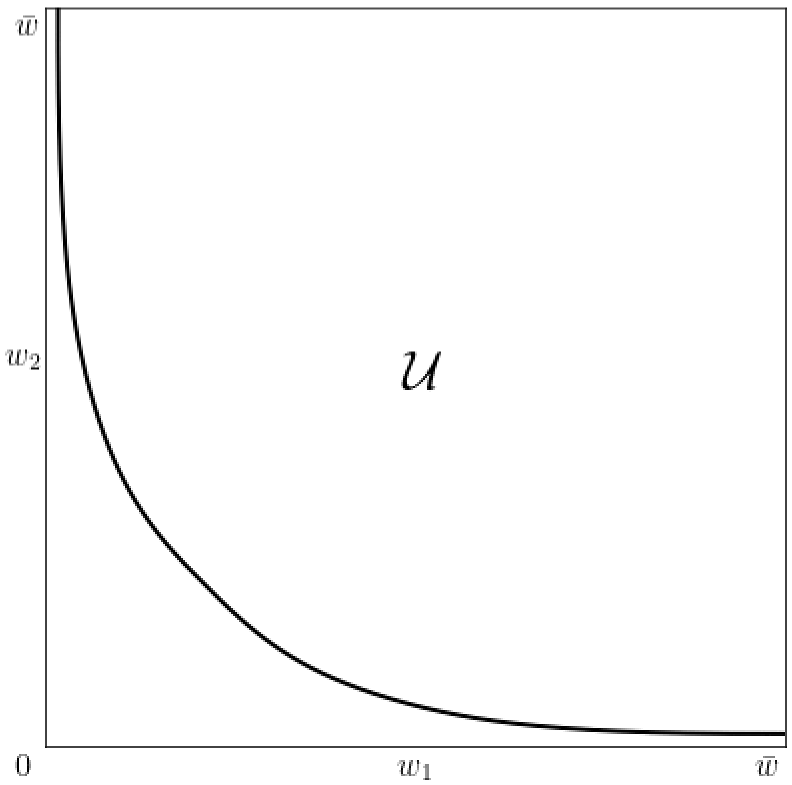
(a) A two-agent case

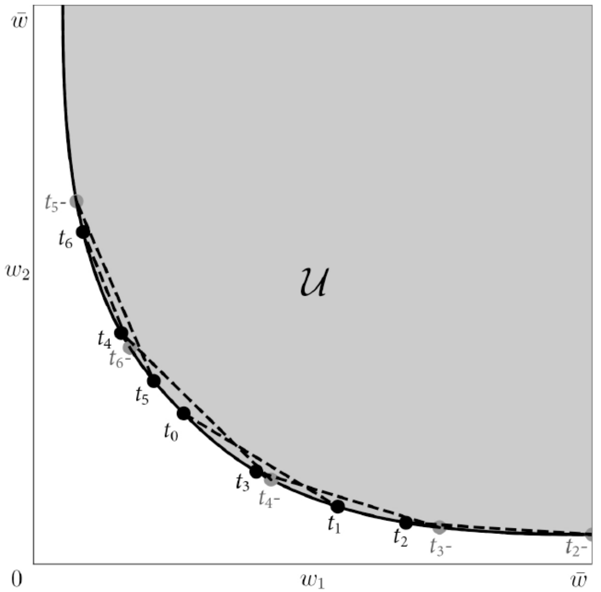


(b) A three-agent case

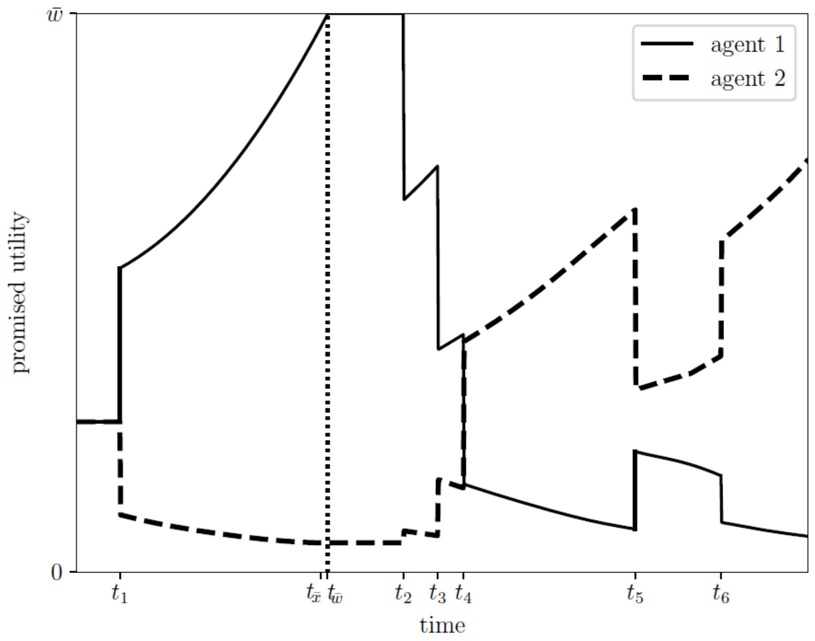
Optimal Boundary EIC Contract



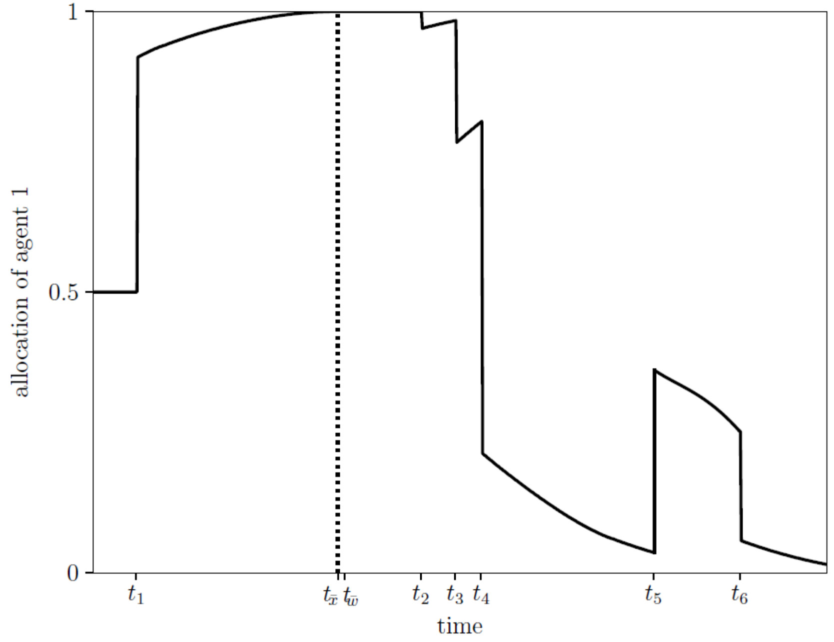




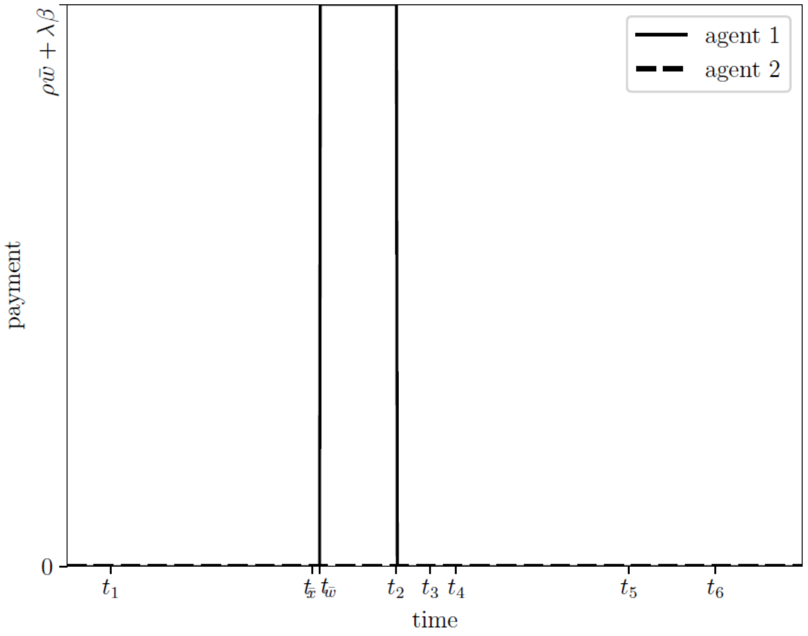
(a) Trajectory of promised utilities on bd($\mathcal{U}$)



(b) Trajectories of promised utilities



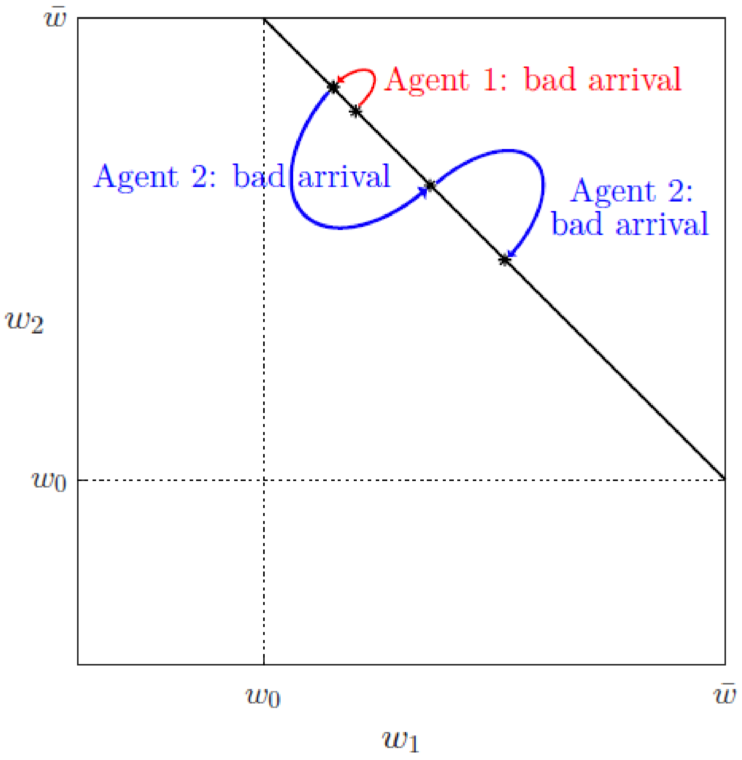
(c) Trajectory of allocations

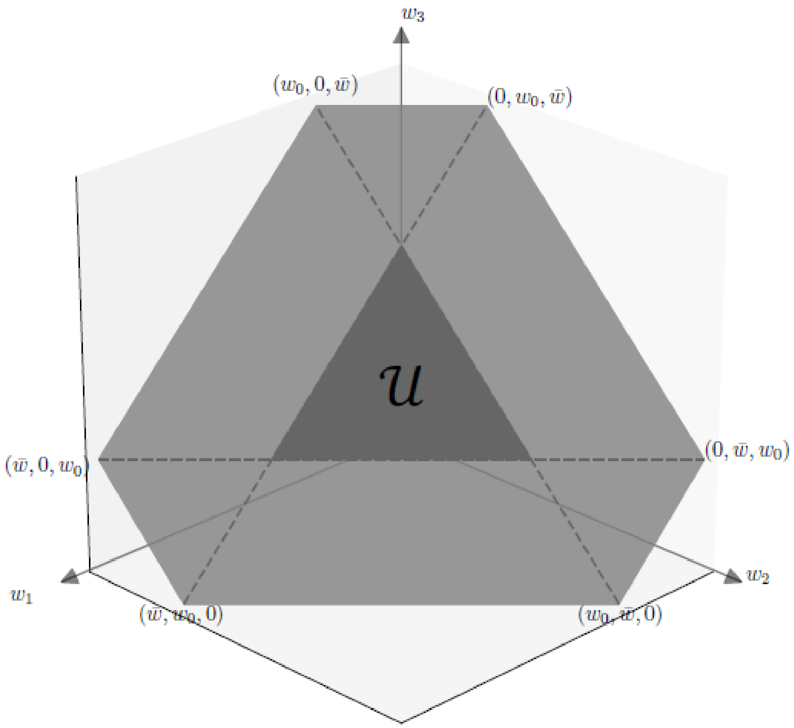


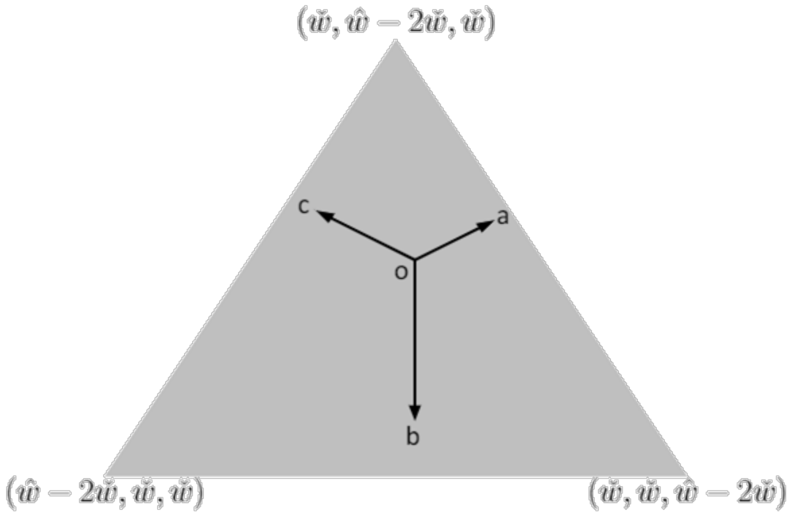
(d) Trajectories of payments

Simple EIC Contract

If $R^a \geq \left(\frac{n}{n-1}\lambda + \rho\right)\beta$

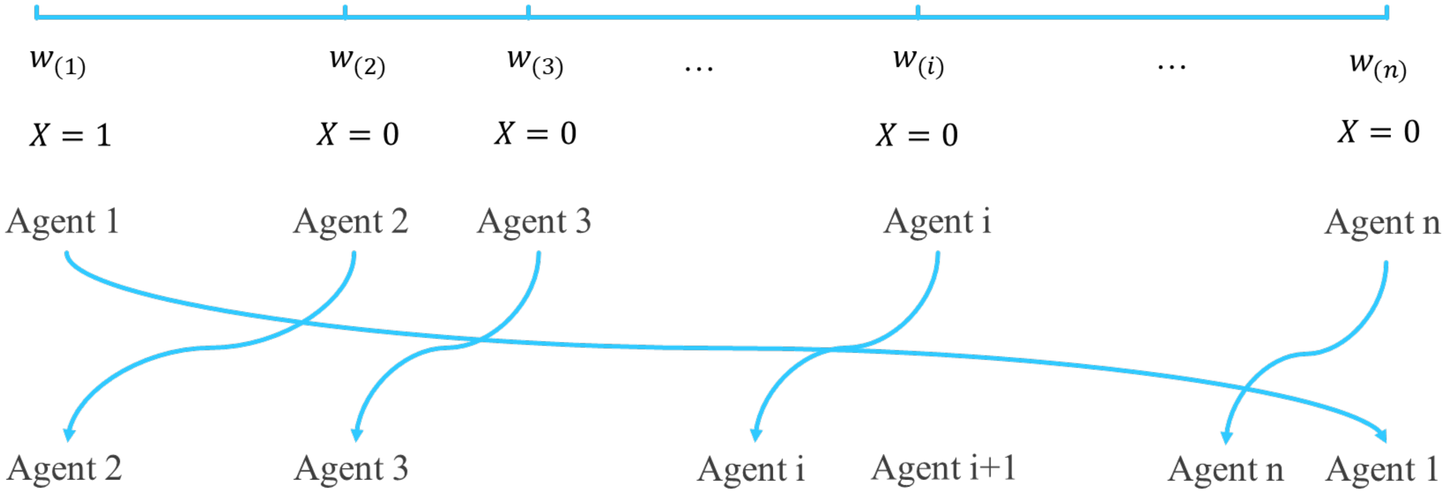






ROTATING CONTRACT

If $R^a \geq \left(\frac{\rho}{1-\tau^{n-1}} + \lambda\right)\beta$

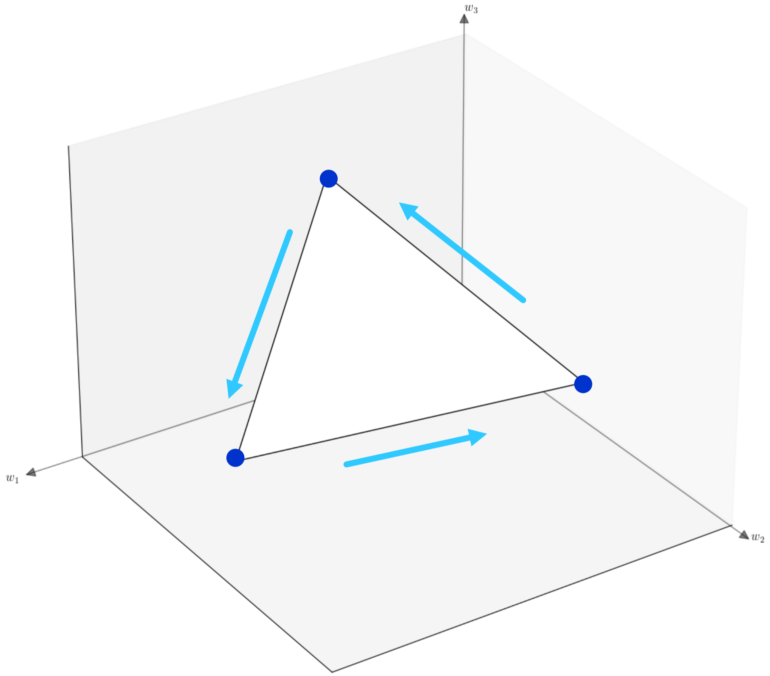


$w_{(1)}$ $w_{(2)}$ $w_{(3)}$ $\dots$ $w_{(i)}$ $\dots$ $w_{(n)}$

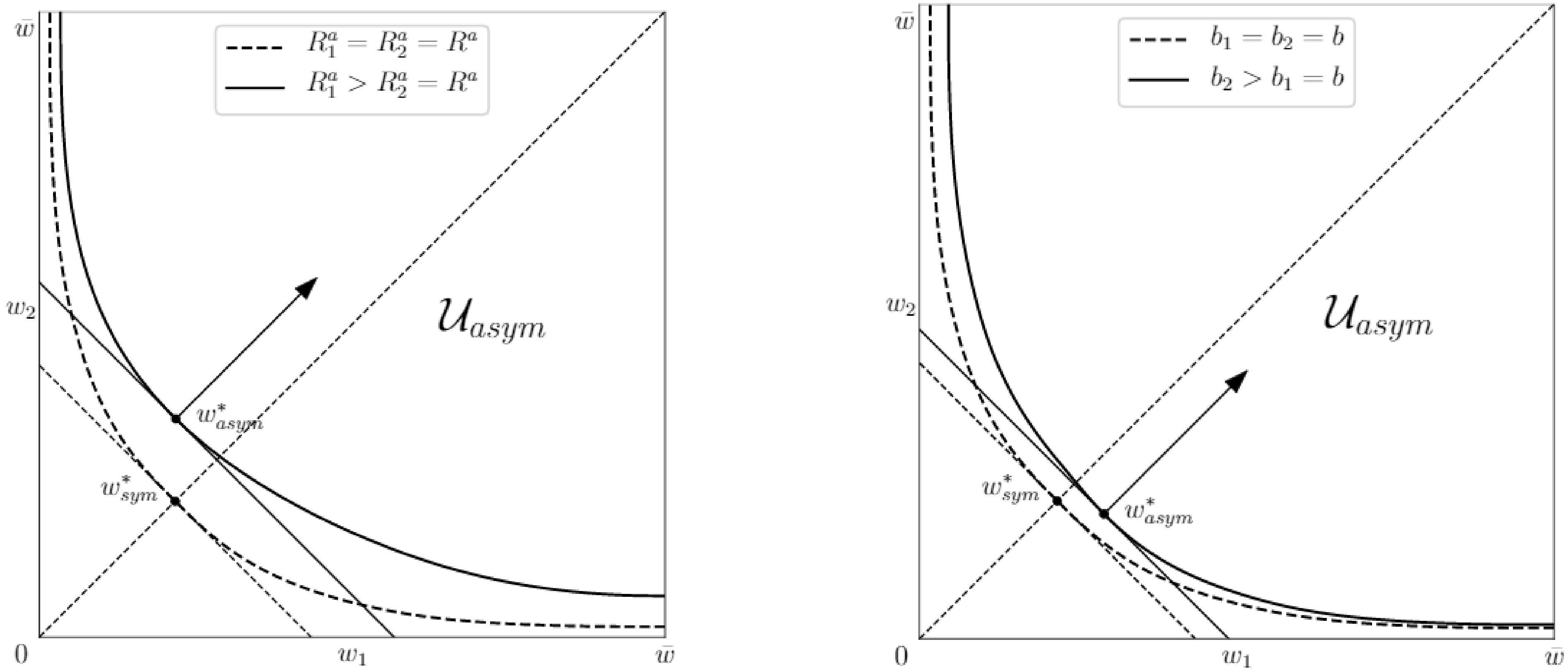
$X = 1$ $X = 0$ $X = 0$ $X = 0$ $X = 0$

Agent 1 Agent 2 Agent 3 Agent i Agent n

Agent 2 Agent 3 Agent i Agent i+1 Agent n Agent 1



ASYMMETRIC AGENTS



(a) $R_1^a > R_2^a$

(b) $b_2 > b_1$

Main takeaway

- Multiple agents, Dynamic moral hazard, Poisson bad arrival.
  - Resource allocation
- Self-generating set and iterative approach
- Optimal EIC contract
  - Not possible for single agent
- Simple EIC contract